Exam. Code : 211001
Subject Code : 4953

M.Sc. Mathematics 1st Semester (Batch 2021-23)

COMPLEX ANALYSIS

Paper—MATH-552

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) If $f(z) = u + iv$ is an analytic function of $z = x + iy$, then show that u and v both are harmonic functions. Also prove that $u = y^3 - 3x^2y$ is a harmonic function.

   (b) Derive the necessary and sufficient condition for $f(z)$ to be analytic in polar co-ordinates.

2. State and prove Cauchy's theorem. Also verify it for $f(z) = z^3 - iz^2 - 5z + 2i$, if path is a circle given by $| z - 1 | = 2$.

SECTION—B

3. (a) State and prove Poisson's integral formula.

   (b) State and prove Liouville's theorem.

4. (a) Define cross ratio. Show that bilinear transformation can be considered as combination of the transformation of translation, rotation, stretching and inversion.

(b) State and prove Schwarz's reflection principle.

SECTION—C

5. (a) If R_1 and R_2 are the radii of convergence of the power series $\Sigma a_n z^n$ and $\Sigma b_n z^n$ resp. then show that the radius of convergence of the power series $\Sigma a_n b_n z^n$ is $R_1 R_2$. Also find the radius of convergence of $\sum \frac{n!}{n^n} z^n$.

(b) State Laurent's theorem and prove its uniqueness.

6. (a) If f(z) is analytic inside and on a simple closed curve c except for a finite number of poles inside C and let $f(z) \neq 0$ on C. Prove that

$$\frac{1}{2\pi i} \int_C \frac{f'(z)}{f(z)} dz = N - P$$

where N and P are respectively the number of zeros and the number of poles of f(z) inside C. A pole or zero of order n is counted n times.

(b) State Rouche's theorem and use it prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

SECTION—D

7. (a) Define isolated and removable singularity. Show that a function which has no singularity in the finite part of the planes or at infinity is constant.

(b) Find residue of

(i) $\frac{1}{(z^2+1)^3}$ at $z = i$

(ii) $\frac{z^3}{z^2-1}$ at $z = \infty$.

8. (a) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)(x^2+9)^2}$.

(b) By contour integration, prove that

$$\int_0^{\infty} \frac{\log(1+x^2)dx}{1+x^2} = \pi \log 2.$$